A Multi-agents Contractual Approach to Incentive Provision in Non-cooperative Networks

Li Lin[1], Jinpeng Huai[1], Yanmin Zhu[2], Chunming Hu[1], and Xianxian Li[1]

[1] Beihang University, Beijing, China
[2] Imperial College London, UK

Abstract. Recent years have witnessed the increasing importance of exchanging information over computer networks or distributed systems. Two end nodes wishing to communicate often rely on independent intermediate nodes to relay messages. In consideration of the rational nature of both the end nodes and intermediate nodes, we have to accommodate two inherently coexistent games: one played between the end nodes and the intermediate nodes and the other played among the intermediate nodes. This is particularly challenging due to the well-known hidden information and the hidden action issues. In this paper we propose a holistic approach PMAC to address the two games, exploiting the principal and multi-agents model creatively. In PMAC, the end nodes make contracts with each intermediate node. The contracts together produce good system properties which are twofold. First, it is guaranteed that the utility of the end nodes is maximized. Second, it is proved that the cooperation of the intermediate nodes can be induced since there exists a Nash equilibrium for the intermediate nodes. However, one serious issue that there may be other Pareto superior Nash equilibriums inevitably hinders the unique implementation of the contracts. We also adopt technique without incurring any additional cost to the end nodes. By knocking out the other redundant Nash equilibriums in the intermediate nodes' game, we ensure that the equilibrium most desired by the end pair is successfully achieved.

Keywords: Non-cooperative network, incentive contracts, hidden-action, hidden-information, mechanism design, collusion, unique implementation.

1 Introduction

It has been ever increasingly important today to exchange information over computer networks or distributed systems. Conventional designs of communication protocols assume that nodes would behave as instructed. However, it is not true in many realistic distributed systems, such as mobile ad hoc networks and the open Internet [1,11] . Two end nodes wishing to communicate highly rely on other intermediate nodes owned by different users. These users are typically rational (or strategic) since each of them always chooses such a way to behave

R. Meersman and Z. Tari (Eds.): OTM 2008, Part I, LNCS 5331, pp. 231–248, 2008.

that its own utility is maximized [1,2,3,4,5,6,7]. This indicates it is hardly possible that intermediate nodes would volunteer to forward data traffic for other nodes because significant cost, such as power and bandwidth, would be incurred. Therefore, it is indispensable for end nodes to provide intermediate nodes with adequate incentive in order to accomplish the data communication task.

Many attempts have been made to employ the notion of reputation management [2,6,8,9,10] to encourage cooperation among nodes. For example, in [9], if a node does not forward other's traffic, it will be considered as uncooperative, which is propagated throughout the network and leads him to be punished in the future. Such reputation systems suffer three major drawbacks. First, it is not trivial to set the initial value of reputation for newcomers. High initial values may make newcomers laze while the low one will discourage them. Second, there is no formal analysis of incentive type provided by such systems. Third, most of them do not consider a serious issue that selfish nodes may collude with each other (e.g. to raise mutual reputation) in order to maximize their welfares.

In contrast, significant progress has been made by recent works [1,3,4,5,7,11,12] exploiting game theoretical approaches. End nodes compensate a relying node with a monetary value that is at least its actual relay cost. This paper focuses on approaches based on game theory. In order to accomplish the data communication task, the end nodes must accommodate two coexistent games.

The **first game** is played between the end nodes and the intermediate nodes. The end pair tries to maximize its utility that is defined to be the benefit of rapid data delivery minus the incentive payment to the intermediate nodes. However, the intermediate node each tries to maximize its profit in terms of the payment received minus the cost of data transfer. It is apparent that the objective of the end nodes is contradictory to that of the intermediate nodes. The **second game** is played among the intermediate nodes. The reward of a node's effort inevitably depends on behaviors performed by other nodes. This suggests that every node has to consider possible actions taken by other nodes when it is making its own decision. Such two games are inherently correlated and must be addressed in a holistic approach. The end pair has to design an incentive scheme that can *maximize its own utility* and at the same time *induce cooperation* of the intermediate nodes.

It has been a great challenge to solve the two coexistent games. In addition to the rational nature of both the end nodes and intermediate nodes, there are two additional difficult issues in distributed systems, known as *hidden information* and *hidden action* [13]. Intermediate nodes have private information, such as transit cost, that is **hidden** from the end nodes. The end nodes typically have no means to have access to such private information of intermediate nodes, which is amplified by the dynamic nature of the environment. Transit costs may vary, depending on the environmental condition of an intermediate node, such as traffic load. As a result, in the first game, *how does the end pair allocate payments optimally in the absence of truthful cost or environmental information*? The result of data delivery is the sequence of the joint efforts of all intermediate nodes. However, the action taken by an intermediate node is also **hidden** from the end nodes. For a delayed data delivery, the end nodes cannot attribute this delay to a specific node, that means, one or even several intermediate nodes together have the opportunity to free ride on others. Then, in the second game, *how can the payment scheme offered by the end pair induce cooperation*?

There is no successful existing work that can address the aforementioned problem holistically. Most of existing works focus either on the routing component or on the forwarding component [14]. The routing component is responsible for determining a cost-efficient path from the source node to the destination node. Proposed works [1,3,11,12] for the routing component partially solves the coexistent games in the sense that they consider the hidden information issue and encourage intermediate nodes to truthfully reveal their costs. While the problem of hidden information is well studied, the utility of the end pair is neglected and the concern of hidden action is not covered in these works. Existing works [4,5,7] for the forwarding component fail to solve the whole problem, either. They only consider the hidden action issue and try to verify that forwarding does happen. The recent work [13] provides valuable findings which consider both the hidden information and the hidden action issues. The paper also points out that there can be the collusion problem. However, it does not propose solutions to this problem.

In this paper, we propose a holistic approach to address the challenging two coexistent games. Exploiting the economic instrument of the *principal and multi-agents model*, we propose a multi-agents contractual approach called PMAC. The end pair is the principal and each intermediate node act as an agent. Before the data communication begins, the principal made contacts with each agent. The set of contracts have very graceful properties. First, it is guaranteed that the utility of the end nodes is maximized if the contracts are executed. Second, it is proved that the cooperation of the intermediate nodes can be induced since following the contracts results in a Nash equilibrium for the intermediate nodes. In addition, our approach explicitly considers the intrinsic dynamics of the environment. Obviously, the income of an intermediate node is highly dependent on the environmental condition. The same effort made by a node is more productive in a good condition than in a poor one. To reflect this reality, we model the environmental conditions as a binary state. The contracts enable intermediate nodes to react to different environmental conditions and thus offset the risk caused by environmental changes.

Although the contractual instrument is powerful, our approach still faces a grand challenge. There may be other Nash equilibrium in the intermediate nodes' game, which may be Pareto superior than the Nash equilibrium desired by the end pair. This raises the serious issue of collusion. The multiple equilibriums create the chance for the intermediate nodes to coordinate and cheat together, which eventually damages the objective of the end pair. We adopt a technique to overcome this issue. It is proved that the end nodes can use this technique to knock out unwanted equilibrium and stop intermediate nodes from collusion without incurring any additional cost to the end pair.

The paper proceeds as follows. In Section 2, we mention related work. In Section 3, we introduce our system model and symbols used in this paper. We give an overview of our approach in Section 4. Detailed design of our approach is presented in Section 5 and Section 6. Our conclusion and future work are in Section 7.

2 Related Work

There have been a lot of research efforts that deal with the selfish issue for message relaying, which can roughly divided into two classes - reputation-based and economic payment-based.

Reputation-based mechanisms [2,6,8,9,10] keep track of a node's history by monitoring its past actions and calculate its reputation based on its transactions with other nodes. With the derived reputation information, other nodes are able to decide whether to forward messages from that node. However, reputation-based systems may be limited by their drawbacks as pointed out in Section 1.

The other class of approaches is to implement pricing mechanisms. In these mechanisms, a monetary amount is charged to nodes causing traffic, while nodes helping forward messages are rewarded. Most approaches in this class fall into two categories, *routing* and *forwarding*. Routing-oriented mechanisms determine a message relaying path from a source to a destination and also determine the payment of a node on the path will receive after forwarding a message. One of the most referenced is VCG mechanisms [1,12] that induce truthful revelation of transfer costs. They solve the problem of hidden information successfully, but fail to take the utility of the end pair into account. Furthermore, most of them neglect the problem of hidden-action. Forwarding-oriented mechanisms [4,5,7] focus on verifying forwarding does happen. For example, Sheng Zhong et al. [7] present a method of reporting by successors. By modeling reporting behaviors as multi-agents game, the mechanism show that honest reporting behaviors is the dominant equilibrium in the game among intermediate nodes, even when a collection of the selfish nodes collude. In this paper, we are concerned with a more realistic scenario where each intermediate node may have more action choices since they reside in an unpredictable changing environment (we model it as a two-state world). As shown in [15], all incentive mechanisms considering more than two states may suffer from the collusion problem in which some other equilibrium generates higher payoffs than the equilibrium desired by the designer.

There are also some works which jointly consider both the routing and the forwarding components. Sheng Zhong et al. [14] present a cooperation-optimal protocol which based on improved VCG mechanisms and cryptographic techniques. But this work may have serious deficiencies with respect to practical applicability particularly because of its high control overhead. In addition, Michael Feldman et al. [13] also use the principle and multi-agents model to study the design of incentive contracts. They address both hidden-action and hidden-information problem. However, the paper does not address the settings where the transit costs of an intermediate node may be highly dependent on the environmental condition.

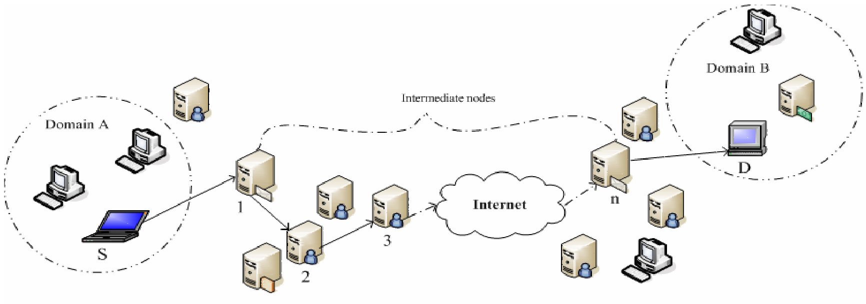

Fig. 1. The nodes of a forwarding path under the consideration of ***S*** and ***D***

In our approach PMAC, the end pair provision sufficient incentive to encourage intermediate nodes to forward messages. Based on the tailor-made principle and multi-agents model [16,17], we propose a provably method to design the optimal incentive contracts of intermediate nodes. Furthermore, we address the problem of multiple equilibriums in the game played among intermediate nodes, which may arise under the proposed payment schemes.

3 System Model

In this paper, we consider a popular and reprehensive application scenario, as showed in Fig. 1. A pair of nodes, S and D, cannot communicate directly and must rely on other intermediate nodes to relay messages. The end pair consider a path of nodes, denoted as $1, 2, \cdots, n$. The multi-hop path can be determined using certain routing protocol, such as AODV [18] in mobile ad hoc network etc. In this paper, we assume that the Quality of Service (QoS) in terms of time latency is the performance focus of the end nodes.

Each intermediate node may have different actions, providing different levels of transfer effort. This may correspond to a service-differentiated network model where packets are forwarded on a best-effort or a priority basis [19]. The action choice of node i can be represented by a random variable $a(i)$. Each action results in an output, which is the transfer time of a message. We denote $t(i)$ as the output choice of node i, where $t(i)$ is in domain T. To all appearances, a shorter transfer time means a higher output.

Additionally, intermediate nodes can choose their own levels of effort, which is often only observable to itself. We don't force the end pair to find out individual actions of each intermediate node and only assume that the end pair can monitor the output produced by each intermediate node. This can be realized by letting each node append to the data packet two timestamps of receiving the data and sending out the data. The part of timestamp is encrypted using the node's private key.

Each action of node i is associated with a cost, which is determined by its current state (or environmental condition), such as traffic load or capacity etc. We denote the state variable of node i by $s(i)$. For simplicity, we assume that each node has two states. It means that $s(i)$ is with binary support $\{b, g\}$, where

b represents a poor condition and g stands for a good condition. For example, node i resides at state b if it has a very high load and at state g when it has a low load. In general, for the same action, its corresponding cost in the poor state is more than that in the good state.

Node i can privately observe its state $s(i)$ before it decides whether to relay a message for the end pair. Since $s(i)$ isn't publicly observed, an enforceable incentive schemes can only be based on the priori knowledge of the end nodes. In this paper, we assume the end pair is able to acquire some prior knowledge of intermediate nodes. There have been many results on dealing with how to get and store this knowledge (e.g., by learning methods). Let $o_b(i)$ be the probability that node i locates at state b and let $o_g(i)$ be the probability that it locates at state g, thus $o_b(i)+o_g(i) = 1$ apparently satisfied. Meanwhile, Let $q_b(i)$ be the conditional probability that node i is in its poor state while the other intermediate nodes all locate at their poor states, and $q_g(i)$ expresses the conditional probability that the node i locates at its good state while the other intermediate nodes all locate at poor states. In this paper, we assume that all nodes work in a correlated environment. Therefore, for any i, there is $1 > q_b(i) > q_g(i) > 0$.

Any selfish node is economically rational node whose goal is to maximize its own utility. On the one hand, the end nodes wish to maximize its own utility defined as the difference between benefit from the outcome of message relaying and incentive cost. The incentive cost is the sum of payments to all intermediate nodes and the outcome is the total transfer time of a message. On the other hand, every intermediate node makes its profit determined by the obtained payment and the transfer cost. The payment is the amount paid by the end pair and the transfer cost depends on its state and action. In this paper, we consider that all nodes are risk-averse [23]. Hence, the nodes utility can be defined as follows.

Utility of the End Nodes: suppose that $w(\cdot)$ is a decreasing and convex function over T, where $w(t(i))$ is the satisfaction degree of the end nodes about the transfer time of node i, and $p(i)$ is the payment to node i. We call $w(t(i)){-}p(i)$ as the end nodes' utility required from node i. If putting

$$W(t) = \sum_{i=1}^{n} w(t(i)), P = \sum_{i=1}^{n} p(i)$$

where $t = (t(1), t(2), \cdots, t(n))$ is the transfer time vector of node $1,\cdots$, node n. Thus the end nodes' level of utility can be denoted by the formula $W(t) - P$.

Utility of Intermediate Nodes: suppose that $D(\cdot)$ is a function over $T\times\{b, g\}$, where $D(t(i), s(i))$ is the dissatisfaction degree of node i when it produces the transfer time $t(i)$ in its state $s(i)$, and D is decreasing and strictly convex about $t(i)$. Let $U(\cdot)$ be an increasing and strictly concave function over the real domain $\Re$, where $U(p(i))$ is the satisfaction degree of node i to the payment $p(i)$. Thus node i's utility can be computed by $U(p(i)) - D(t(i), s(i))$. In addition, there exists a reservation utility level, $\bar{U}(s(i))$, for node i, which is determined by the state of node i and represents the expected utility that it would obtain if it refuses to sign a contract with the end pair. Since the same action incurs more cost in the poor state than that in the good state, for the same output choice $t(i)$, so $D(t(i), b) > D(t(i), g)$ satisfies.

In consideration of the rational nature of both the end nodes and intermediate nodes, it should be cognizant that there are two coexistent games: one is the game between the end pair and the intermediate nodes; the other is the game among the intermediate nodes. Hence, the objectives of system design are twofold,

- to maximize the utility of the end nodes, and meanwhile
- to induce cooperation of the intermediate nodes so as to accomplish the data transfer task desired by the end nodes.

4 Overview of PMAC

In this section, we give a brief overview of our proposed incentive scheme called PMAC which exploits the principal and multi-agents model. The end nodes are considered as the principal and the intermediate nodes as agents. Before the data communication takes place, the end pair makes **contracts** with the intermediate nodes. Each contract specifies the payment amount the end pair will pay to the intermediate node under different conditions. The basic procedure is depicted in Table 1.

To provide an efficient incentive scheme, PMAC must take into account the rational nature of the intermediate nodes. Each intermediate node always enforces an action that can maximize its own utility. Thus, PMAC must meet two intrinsic constraints posed by the intermediate nodes [16,17]:

- **Individual rationality constraint (IR)**: the expected utility from accepting contract should (weakly) exceed its reservation.
- **Incentive compatibility constraint (IC)**: the expected utility from exerting the action and then producing the transfer time desired by the end pair should (weakly) exceed its expected utility from exerting the other actions.

Initially, the end pair and intermediate nodes all share the same imperfect information about $s(i)$ $(i = 1, \cdots, n)$. The end pair designs a contract for node i in accordance with the corresponding constraint condition IR and IC. Before a contract is agreed upon, however, node i acquire perfect information about the

Table 1. The basic procedure

Step 1:	**the end pair offers each intermediate node a contract;**
Step 2:	**each intermediate node observes its current state and decides whether to accept the contract or not;**
Step 3:	**the end pair sends the message to the first intermediate node;**
Step 4:	**each intermediate node exerts an action to forward the data and produces a transfer time(the game among intermediate nodes)**
Step 5:	**the contracts are performed.**

Table 2. The contract offered to node J

The node can choose to produce transfer time either $t_b(j)$ or $t_g(j)$. The payment given by the end pair $p(j)$ is dependent not only on his transfer time but also on the transfer time produced by node k.

• when the node produces $t_b(j)$, then
If the other produce $t_b(k)$, the payment amount is $p_{bb}(j)$;
If the other produce $t_g(k)$, the payment amount is $p_{bg}(j)$
•when the node produces $t_g(j)$, then
If the other produce $t_b(k)$, the payment amount is $p_{gb}(j)$
If the other produce $t_g(k)$, the payment amount is $p_{gg}(j)$

random variable $s(i)$ that characterizes the current state under which it is employed. Then, after the terms of the agreement are made final, the intermediate node chooses a level of effort to put forth, the transfer time is observed, and it is compensated according to the realized transfer time as per the agreement.

Since neither $s(i)$ nor $a(i)$ is publicly observed by the end pair, an enforceable contract can only be based on the transfer time $t(i)$ of node i. According to $D(t(i), b) > D(t(i), g)$, it is obvious that the transfer time desired by the end pair in the poor state should be different with the one in the good state. In this paper, we assume that, for any i, $t_b(i)$ is the transfer time that the end pair expect node i produce when $s(i) = b$, and $t_g(i)$ is the one when $s(i) = g$.

The latency experienced by the message from S to D is determined by the joint efforts of all intermediate nodes. Therefore, node i's marginal contribution to the latency is dependent on the actions of the other nodes. Thus, the contracts offered to these nodes are interdependent. Table 2, illustrates a typical contract offered to node j. For presentation simplification, we concentrate on the case where the end pair make contracts with two nodes j, k. Nevertheless, the derived results can be generalized to situations with any finite number of intermediate nodes. We shall point out the extensions as necessary.

To address the two coexistent games, PMAC must complete two tasks which are known in Economics literature as mechanism design [20] and unique implementation [21]. Facing strategic agents, the mechanism design addresses the system that should have at least one Nash equilibrium. Whereas, the unique implementation aims to knock out undesired Nash equilibriums and keep the only desired Nash equilibrium.

We first propose our mechanism design for PMAC by designing optimal contracts, as presented in Section 5. In consideration of challenging issues such as information asymmetry and hidden action [22], we formulate the design of optimal contracts as a programming problem. The objective is to maximize the utility level of the end pair while taking into account the IC and IR constraints of the intermediate nodes. We show that the proposed optimal incentive contracts can implement a vector of intermediate nodes' strategies as a Nash equilibrium for the game among the intermediate nodes.

Next, we deal with the unique implementation issue, as discussed in Section 6. We firstly demonstrate, by a concrete example, that there exists another Nash equilibrium which is Pareto superior (from the intermediate nodes' perspectives) than the end pair's preferred equilibrium indeed. We then adopt a technique to ensure the equilibrium preferred by the end nodes: the end nodes use one of the intermediate nodes to police the others by offering it a set of additional strategies to choose. It therefore turns out that there is no need for the end pair to employ the costly solution to strengthen incentive to an intermediate node whose strategies have been artificially constrained to be a dominant one. Instead, the end nodes can ensure that its optimal incentive schemes can be implemented uniquely. To the best of our knowledge, this is the first solution of its kind in message relaying research that does not decrease the utility of the end nodes.

5 Optimal Incentive Contracts

It is enough to consider the optimal contract with respect to one intermediate node, accordingly, we shall leave out the arguments $i = j, k$, whenever so doing does not create confusion. Thus, for any intermediate node, t_b and t_g are transfer time desired by the end pair in the state b and g, respectively. And let $p_{ss'}$ ($s = b, g$; $s' = b, g$) be the payment with respect that the intermediate node produces transfer time t_s while the other produces transfer time t'_s. The IC and IR condition of the node are formulated as follow.

$$q_bU(p_{bb}) + (1 - q_b)U(p_{bg}) - D(t_b, b) \geq \bar{U}(b) \quad (1)$$
$$q_gU(p_{gb}) + (1 - q_g)U(p_{gg}) - D(t_g, g) \geq \bar{U}(g) \quad (2)$$
$$q_bU(p_{bb}) + (1 - q_b)U(p_{bg}) - D(t_b, b) \geq q_bU(p_{gb}) + (1 - q_b)U(p_{gg}) - D(t_g, b) \quad (3)$$
$$q_gU(p_{gb}) + (1 - q_g)U(p_{gg}) - D(t_g, g) \geq q_gU(p_{bb}) + (1 - q_g)U(p_{bg}) - D(t_b, g) \quad (4)$$

Therefore, the optimal contract to one intermediate node (from the end nodes' points of view) can be represented as a solution to the following programming ($E.IP$), where t_b, t_g, p_{bb}, p_{bg}, p_{gb} and p_{gg} are the decision variables.

$$\underset{t_b,t_g,p_{bb},p_{bg},p_{gb},p_{gg}}{\boldsymbol{Max}} \begin{pmatrix} o_b[q_b(w(t_b) - p_{bb}) + (1 - q_b)(w(t_b) - p_{bg})] \\ +o_g[q_g(w(t_g) - p_{gb}) + (1 - q_g)(w(t_g) - p_{gg})] \end{pmatrix} \quad (5)$$

In general, we can prove the following statement, which demonstrates that the end nodes are able to design the optimal contracts for intermediate nodes. Hence, it is guaranteed that the utility of the end nodes is maximized if these contracts are executed.

Proposition 1. *Suppose that each intermediate node privately observes his state $s \in \{b, g\}$ before he signs a contract with the end pair, then the end nodes can design the optimal contracts for it if $q_b > q_g$.*

Proof. The task of this proof is to show that there exists a solution to the programming ($E.IP$). First, we are looking for the values p_{bb}, p_{bg}, p_{gb} and p_{gg}, which satisfy the following conditions:

$$q_bU(p_{bb}) + (1 - q_b)U(p_{bg}) - D(t_b, b) = \bar{U}(b) \quad (1^*)$$
$$q_gU(p_{gb}) + (1 - q_g)U(p_{gg}) - D(t_g, g) = \bar{U}(g) \quad (2^*)$$

and (3),(4)

Putting $U(p_{gb})-U(p_{bb})=\alpha, U(p_{gg})-U(p_{bg})=\gamma$, where $D(t_g,g)-D(t_b,g)\leq \gamma\leq\alpha\leq D(t_g,b)-D(t_b,b)$.

Since

$$\begin{aligned}&q_b(U(p_{bb})-U(p_{gb}))+(1-q_b)(U(p_{bg})-U(p_{gg}))+D(t_g,b)-D(t_b,b)\\&=-q_b\alpha-(1-q_b)\gamma+D(t_g,b)-D(t_b,b)\\&\geq(1-q_b)(\alpha-\gamma)\geq 0\end{aligned}$$

and

$$\begin{aligned}&q_g(U(p_{gb})-U(p_{bb}))+(1-q_g)(U(p_{gg})-U(p_{bg}))+D(t_b,g)-D(t_g,g)\\&=q_g\alpha+(1-q_g)\gamma-(D(t_g,g)-D(t_b,g))\\&\geq q_g(\alpha-\gamma)\geq 0\end{aligned}$$

Therefore, it is straightforward to verify that the inequalities (3) and (4) are satisfied. Thus, substituting for $U(p_{gb})$ and $U(p_{gg})$ in the equalities (1^*) and (2^*), there is an equation group

$$\begin{cases}q_bU(p_{bb})+(1-q_b)U(p_{bg})=D(t_b,b)+\bar{U}(b)\\q_gU(p_{bb})+(1-q_g)U(p_{bg})=\bar{U}(g)+D(t_g,g)-q_g\alpha+(1-q_g)\gamma\end{cases}$$

Because $q_b>q_g$ implies $\begin{vmatrix}q_b & 1-q_b\\ q_g & 1-q_g\end{vmatrix}\neq 0$ so there exists a unique solution to the above equation group. The solution has the character t_b and t_g. Furthermore, for any $(t_b,t_g)\in T^2$, we can always solve for p_{bb},p_{bg}, p_{gb} and p_{gg} by the function $U(\cdot)$, which satisfies the conditions (1^*), (2^*), (3) and (4).

Second, given $(t_b,t_g)\in T^2$, we can compute the value of the Formula (5), in which we substitute for (t_b,t_g) and the corresponding p_{bb}, p_{bg}, p_{gb} and p_{gg}. Comparing these values, among which the maximum one will be found. Thus the program $(E.IP)$ is solved. □

The following proposition exploit certain properties of the optimal contracts. These properties will be helpful for the later discussion. The detail proof can be found in the Appendix.

Proposition 2. *The above optimal contracts have the following properties.*

$(A.)$ $p_{bb}>p_{bg}, p_{gb}=p_{gg}$
$(B.)$ $q_bU(p_{bb})+(1-q_b)U(p_{bg})-D(t_b,b)=\bar{U}(b)$
$(C.)$ $q_gU(p_{gb})+(1-q_g)U(p_{gg})-D(t_g,g)\geq\bar{U}(g)$
$(D.)$ $q_bU(p_{bb})+(1-q_b)U(p_{bg})-D(t_b,b)>q_bU(p_{gb})+(1-q_b)U(p_{gg})-D(t_g,b)$
$(E.)$ $q_gU(p_{gb})+(1-q_g)U(p_{gg})-D(t_g,g)=q_gU(p_{bb})+(1-q_g)U(p_{bg})-D(t_b,g)$

These properties highlight a number of important points. First, as shown by $(B.)$, an intermediate node is held to his reservation level of expected utility when he observes the bad state b. Second, an intermediate node may receive rents when he observes the good state g, as indicated by $(C.)$ and $(E.)$. Especially, $(E.)$ shows that only one of incentive constraints binds at an optimum. Hence, the end pair has to discourage an intermediate node from producing transfer time t_b when it has in fact observed g. Third, $(A.)$ shows that $\{p_{bb}, p_{bg}\}$ can help with incentives. If $q_b > q_g$, the node is aware that when he has in fact observed g, not b, the chance that he will get the higher payment p_{bb} is relative slight, and the chance that he will receive the small payment p_{bg} is relative great. So he will not produce t_b when he has in fact observed g. Fourth, $(D.)$ says that each intermediate node will be induced to produce less transfer time t_b when he observe b.

Proposition 3. *Suppose the optimal incentive contracts of node j and k are designed by solving the corresponding program $E.IP$, then following these contracts results in a Nash equilibrium in the game between node j and k.*

Proof. We denote a strategy by sy^*, which represents that the intermediate node takes t_b to transfer messages when he observes the poor state b, and takes t_g to transfer messages when he observes the good state g. The IC and IR constraints mean exactly that each node has no profitable deviations, this suggests that $NE^* = (sy^*, sy^*)$ should be a Nash equilibrium in the game played between node j and k. □

This proposition shows that the optimal contracts can induce the cooperative behaviors of the intermediate nodes.

6 Unique Implementation of Optimal Incentive Contracts

As revealed by Proposition 1 and 3, the optimal incentive contracts can results in a Nash equilibrium in the game among intermediate nodes, whose outcome maximizes the end nodes' utility. However, these contracts may fail to guarantee the end pair's desired equilibrium as the unique equilibrium. In this section, we address the unique implementation problem of the optimal contracts.

6.1 Collusion Problem

The problem of unique implementation is most explicit when intermediate nodes are assumed to use Nash equilibrium strategies, since typically there is multiple Nash equilibrium. Now, we demonstrate that there exists another Nash equilibrium which is Pareto superior than the end pair's preferred equilibrium NE^* indeed. It is then not at all clear that NE^* will arise. We consider what happen if one of intermediate nodes always produces the transfer time t_b whether he resides the poor or good state not. We denote this strategy by sy^0.

Proposition 4. *Suppose the conditions cited in Proposition 2 hold, and let $NE^0 = (sy^0, sy^0)$. Then, NE^0 is also a Nash equilibrium in the game between node j and k. And both node j and k will prefer NE^0 to NE^*.*

Proof. For node j, we suppose the condition that node k always chooses sy^0 is satisfied. According to $(A.)$ and $(E.)$ in Proposition 2, then, when node j observes the poor state b, there is

$$\begin{aligned} U(p_{bb}) - D(t_b, b) &> q_b U(p_{bb}) + (1 - q_b) U(p_{bg}) - D(t_b, b) \\ &\geq q_b U(p_{gb}) + (1 - q_g) U(p_{gg}) - D(t_g, b)) \\ &= U(p_{gb}) - D(t_g, b) \end{aligned}$$

when he observes the good state g, there is

$$\begin{aligned} U(p_{bb}) - D(t_b, g) &> q_g U(p_{bb}) + (1 - q_g) U(p_{bg}) - D(t_b, g) \\ &= q_g U(p_{gb}) + (1 - q_g) U(p_{gg}) - D(t_g, g)) \\ &= U(p_{gb}) - D(t_g, g) \end{aligned}$$

Hence, sy^0 is the best response of node j under this condition. This argument works symmetrically for node k. NE^0 is therefore a Nash equilibrium. In addition, we know that all intermediate nodes will receive stochastic payment if they play as NE^*. But if they play as the equilibrium NE^0, they both has the payment p_{bb} with certainty. Hence they will prefer NE^0 to NE^*. □

The states and actions of the intermediate nodes are often hidden from the end pair, so they may coordinate and cheat together to take low-effort action and produce slower transfer time in both states. Of course, the end pair will be strictly worse off. The general approach for knocking out multi equilibriums is to strengthen the incentive constraints of one player to make his choices a dominant strategy for it, similar to that adopted in [15]. Although this kind of method does guarantee a unique equilibrium, it is also costly since the end pair has to strengthen the incentive constraints in $(E.IP)$. In this paper, we adopt an alternative, costless method to address this issue in non -cooperative networks, which is based on the idea of extending strategies space [23,24]. We will show that the end pair can guarantee NE^* as a unique equilibrium.

6.2 A Unique-Equilibrium Mechanism

We consider the following mechanism, which uses one intermediate node to regulate the other by offering it a set of additional strategies, but in fact these additional strategies will not affect the final equilibrium.

The end pair select one node, say node j, and then offer node j a range of extra options denoted by $t_b(j)(\theta)$, where $0 < \theta < 1 - q_b(j)$. If node j produces $t_b(j)(\theta)$, then it essentially spends $t_b(j)$ on transferring a message. Here θ acts as a signal that node j sends to the end pair. That means, from node j's perspective, the probability that node k is choosing to produce transfer time $t_b(k)$ is at least $q_b(j) + \theta$."

When the end nodes receive a signal from node j and node k spends $t_b(k)$ on transferring messages indeed, they will pay node k the certainty equivalent of the lottery $\bar{P}_b(k)$, which node k would face if he had observed the state g.

However, if node k actually produces $t_g(k)$ and node j signals some θ by choosing $t_b(j)(\theta)$, then node k should be compensated by receiving a higher

Table 3. The reward and punishment scheme for signaling θ

The time that node k spends on transferring messages	**In return for choosing $t_b(j)(\theta)$, node j is paid**
$t_b(k)$	$p_{bb}(j) + \xi(\theta)$
$t_g(k)$	$p_{bb}(j) - \psi(\theta)$

payment denoted by $p_{gb}(k) + \eta$. Obviously, the increase $\eta > 0$ cannot be too great, because too high a compensation might admit unwanted equilibrium. We leave two upper bounds on η in the proof Proposition 5.

It must be the case that node j has an incentive to exercise one of the options $t_b(j)(\theta)$ if node k is choosing to produce $t_b(k)$ more often than he would in equilibrium NE^*. Meanwhile, node j should be punished if he sends signals maliciously. Table 3 shows the reward and punishment scheme for signaling θ in PMAC, where the functions ξ and ψ are both strictly positive for $0 < \theta < 1 - q_b(j)$, and satisfy

$$\begin{aligned}(q_b(j) + \theta)U(p_{bb}(j) + \xi(\theta)) + (1 - q_b(j) - \theta)U(p_{bg}(j) - \psi(\theta))\\ = (q_b(j) + \theta)U(p_{bb}(j)) + (1 - q_b(j) - \theta)U(p_{bg}(j))\end{aligned} \tag{6}$$

The Formula (6) shows that if node j signals θ, it will gain the expected payments equivalent of the lottery under the optimal contract. It suggests that the scheme for signaling does not introduce any additional cost to the end pair.

The idea of the scheme is as follows. For node j, the difference in expected utility between choosing $t_b(j)(\theta)$ and $t_b(j)$ can be defined by the following formula.

$$\begin{aligned}\triangle^\theta(q) = qU(p_{bb}(j) + \xi(\theta)) + (1 - q)U(p_{bg}(j) - \psi(\theta)) - D(t_b(j), b)\\ -[qU(p_{bb}(j)) + (1 - q)U(p_{bg}(j)) - D(t_b(j), b)]\end{aligned}$$

Then ξ and ψ are both positive implies $\frac{\partial(\triangle^\theta(q))}{\partial q} > 0$, and (6) implies $\triangle^\theta(q_b(j) + \theta) = 0$. It follows that $\triangle^\theta(q) > 0$ for $q > q_b(j) + \theta$, both $\triangle^\theta(q_b(j)) < 0$ and $\triangle^\theta(q_g(j)) < 0$ for all $\theta > 0$. Therefore, on the one hand, for any $\theta(0 < \theta < q - q_b(j))$, node j prefers $t_b(j)(\theta)$ to $t_b(j)$. That is, if node j has observed b and also assesses that node k is choosing $t_b(k)$ more than he would in equilibrium NE^*, it will send a signal to the end pair. On the other hand, if node k is in fact choosing as in equilibrium NE^*, the above scheme will not make node j have an incentive to signal some $\theta > 0$.

Additionally, an issue the end pair must consider is how much should an intermediate node be paid for producing transfer time $t_s(i)(s = b, g; i = j, k)$ if the other refuses to sign his contract. Rather than give the details here of how the end pair can do this, we show them in Table 4, which displays the full payments of node j and node k.

The following proposition indicates that the above mechanism can implement the optimal contracts uniquely and incur no additional cost of the end pair. The proof appears in the Appendix.

Proposition 5. *Suppose the payments of node j and node k are given in Table 4. Then the optimal contracts can be implemented as the unique equilibrium NE^*.*

Table 4. The payment matrix of node J and K

Choices	$t_b(k)$	$t_g(k)$	Refusal
$t_b(j)$	$(p_{bb}(j), p_{bb}(k))$	$(p_{bg}(j), p_{gb}(k))$	$(p_{bb}(j), -)$
$t_b(j)(\theta)$	$(p_{bb}(j) + \xi(\theta), \bar{p_b}(k))$	$(p_{bb}(j) - \psi(\theta), p_{gb}(k) + \eta)$	$(p_{bb}(j) + \xi(\theta), -)$
$t_g(j)$	$(p_{gb}(j), p_{bg}(k))$	$(p_{gg}(j)), p_{gg}(k))$	$(p_{gg}(j) - \eta, -)$
Refusal	$(-, p_{bb}(k))$	$(-, p_{gb}(k) - \eta)$	$(-, -)$

It seems a problem how to select the node to send the signal, especially when there are more than two intermediate nodes. However, the previous discussion suggests the node sending the signal will have a lower benefit if it behaves maliciously. Therefore, we can randomly select a node to act as the signalling node.

7 Conclusion and Future Work

In this paper we have studied the problem of provisioning incentive where rational end nodes rely on rational intermediate nodes to accomplish the data communication task. We have identified two challenging coexistent games that exist in this problem setting. Our approach PMAC employs the principal and multi-agents model and holistically accommodates the two games. It recognizes the fact that the network environment is constantly changing and the benefit of an intermediate node is highly dependent on the current environmental condition. By making contracts with each intermediate node, the end nodes maximize their utility. Meanwhile, the contracts can induce cooperation from the intermediate nodes, which is proved by showing that there exists a Nash equilibrium. In addition, the contracts also allow intermediate nodes to choose the best action under different environmental conditions. In response to the collusion issue that prohibits the unique implementation of the contracts, PMAC employs one of the intermediate nodes to regulate other nodes. The feature of this technique is that it does not introduce additional cost to the end nodes.

However, some problems are not covered in this paper. First, A limitation remains that it only considers binary states, while in a real world a node can have more states. We also need to study other incentive mechanisms when the stochastic structure of state is richer. Second, we will explore the mechanism which can be uniquely implemented, in settings where the end nodes haven't the topology as a priori information. Finally, our ongoing research will test the performance of PMAC under different multiple-hop path settings.

Acknowledgement

This work is partially supported by grants from the China 863 High-tech Program (Project No. 2007AA01Z426), China 973 Fundamental *R&D* Program (No. 2005CB321803) and National Natural Science Funds for Distinguished Young Scholar (Project No. 60525209). We would like to thank the anonymous reviewers for their constructive comments and suggestions.

References

1. Anderegg, L., Eidenbenz, S.: Ad hoc-VCG: a truthful and cost-efficient routing protocol for mobile ad hoc nerworks. In: ACM MOBICOM (2003)
2. Buchegger, S., Boudec, J.-Y.L.: Performance analysis of the CONFIDANT protocol: Cooperation of nodes. In: ACM MOBIHOC (2002)
3. Buttyan, L., Hubaux, J.P.: Stimulating cooperation in self-organizing mobile ad hoc networks. ACM Journal for Mobile Networks, special issue on Mobile Ad Hoc Networks 58 (2002)
4. Jakobsson, M., Hubaux, J.P., Buttyan, L.: A micropayment scheme encouraging collaboration in multi-hop cellular networks. In: Wright, R.N. (ed.) FC 2003. LNCS, vol. 2742. Springer, Heidelberg (2003)
5. Mahajan, R., Rodrig, M., Wetherall, D., Zahorjan, J.: Experiences applying game theory to system design. In: ACM SIGCOMM Workshop on Practice and Theory of Incentives and Game Theory in Networked Systems (2004)
6. Molva, R., Michiardi, P.: Core: A collaborative reputation mechanism to enforce node cooperation in mobile ad hoc networks. Technical Report, Institute Eurecom Research Report (2001)
7. Zhong, S., Chen, J., Yang, Y.R.: Sprite: A Simple, Cheat-Proof, Credit-based system for Mobile Ad-hoc Networks. In: IEEE INFOCOM (2003)
8. Marti, S., Giuli, T.J., Lai, K., Baker, M.: Mitigating routing misbehaviour in mobile ad hoc networks. In: ACM MOBICOM (2000)
9. Blanc, A., Liu, Y.K., Vahdat, A.: Designing Incentive for Peer-to-Peer Routing. In: IEEE INFOCOM (2005)
10. Yanbin, L., Yang, Y.R.: Reputation propagation and agreement in mobile ad-hoc networks.In: IEEE WCNC (2003)
11. Feigenbaum, J., Papadimitriou, C., Sami, R., Shenker, S.: A BGP-based Mechanism for Lowest-Cost Routing.In: ACM PODC (2002)
12. Weizhao, W., Xiang-Yang, L.: Low-Cost Routing in Selfish and Rational Wireless Ad Hoc Networks. IEEE Transactions on Mobile Computing 5, 596–607 (2004)
13. Feldman, M., Chuang, J., Stoica, I., Shenker, S.: Hidden-Action in Multi-Hop Routing. In: ACM EC (2005)
14. Zhong, S., Li, L.E., Yanbin, L., Yang, Y.R.: On Designing Incentive Compatible Routing and Forwarding Protocols in Wireless Ad Hoc Networks - an Integrated Approach Using Game Theoretical and Cryptographic Techniques. In: ACM MOBICOM (2005)
15. Jurca, R., Faltings, B.: Collusion-resistant, Incentive-compatible Feedback Payments. In: ACM EC (2007)
16. Demski, J.S., Sappington, D.: Oprimal Incentive Contracts with Multiple Agents. Journal of Economic Theory 33, 152–171 (1984)
17. Mookherjee, D.: Optimal Incentive Schemes with Many Agents. Review of Economic Studies 51 (1984)
18. Johnson, D.B., Maltz, D.A., Hu, Y.C., Jetcheva, J.G.: The dynamic source routing protocol for mobile ad hoc network (DSR). IETF Internet Draft draft-ietf-manet-dsr-07.txt (2002)
19. Blake,S., Black,D., Carlson,M.,Davies,E., Wang, Z., Weiss,W.: An Architecture for Differentiated Service. RFC2475 (1998)
20. Nisan, N., Ronen, A.: Algorithmic mechanism design. Games and Economic Behavior 35, 166–196 (2001)

21. Jachson, M.O.: A Crash Course in Implementation Theroy. Social Choice and Welfare 18, 655–708 (2001)
22. Holmstrom, B.: Moral Hazard in Teams. Bell Journal of Economic 13, 324–340 (1982)
23. Ma, C.-T.: Unique Implementation of Incentive Contracts with Many Agents. Review of Economic Studies 55, 555–572 (1988)
24. Ma, C.-T., Moore, J., Turnbull, S.: Stopping agents from cheating. Journal of Economic Theory 46, 355–372 (1988)

Appendix

The Proof of Proposition 2

Solving the group of equations (1^*) and (2^*), it is apparent that there is $U(p_{bb})-U(p_{bg})>0$. And U is increasing, so $p_{bb}>p_{bg}$. Consider the first-order conditions of $(E.IP)$ with respect to p_{gb}, p_{gg}:

$$\begin{aligned} -o_g q_g + \mu_1 q_g U'(p_{gb}) + \mu_2 q_g U'(p_g b) &= 0 \\ -o_g(1-q_g) + \mu_1(1-q_g)U'(p_{gg}) + \mu_2(1-q_g)U'(p_{gg}) &= 0 \end{aligned}$$

where μ_1 and μ_2 are multipliers of the Formula (2) and (4). Since it cannot be the case that both μ_1 and μ_2 are zero, these first-order conditions give $U'(p_{gb}) - U'(p_{gg}) = \frac{p_g}{\mu_1+\mu_2} - \frac{p_g}{\mu_1+\mu_2} = 0$, that is $U'(p_{gb}) = U'(p_{gg})$. U is increasing and strictly concave implies $p_{gb} = p_{gg}$. So the property $(A.)$ satisfies.

Since the IR constraint (2), the property $(C.)$ satisfies obviously.

Let $p_{gb} = p_{gg} = p_g$, consider the first-order conditions of $(E.IP)$ with respect to p_{gb}, p_{gg} and p_g. Due to paper space limitation, we don't list these conditions in detail. We denote the multipliers of the Formula (1), (2) and (4) by μ_3, μ_4 and μ_5, respectively. Note that $\mu_3 > 0$. For if not, then by complementary slackness (1) holds as a strict inequality, and the end pair can reduce p_{bb} and p_{bg} without violating (4). This is a contradiction. So the property $(B.)$ satisfies.

Next, we will show that (4) binds with a positive multiplier μ_5, that is, the property $(E.)$ satisfy. If $\mu_5 = 0$, we only need to worry about the two IR constraints and the end pair can attain the optimal $*$, where $p_{bb} = p_{bg} = {p_b}^*$ and also $U({p_b}^*) - D({t_b}^*, b) = \bar{U}(b) = U({p_g}^*) - D({t_g}^*, g) = \bar{U}(g)$. But $U({p_b}^*) - D({t_b}^*, g) > U({p_b}^*) - D({t_b}^*, b) = \bar{U}(g) = U({p_g}^*) - D({t_g}^*, g)$ so that (4) is violated, thus it must be that $\mu_5 > 0$. So the property $(E.)$ satisfies.

Now write the optimal contract the end pair offers one intermediate node as $\{C_b, C_g\} = \{(t_b, p_{bb}, p_{bg}), (t_g, p_{gb}, p_{gg})\}$ and define $\tau_s = w(t_s) - q_s p_{sb} - (1-q_s)p_{sg}$, for $s = b, g$. (Note that $p_{gb} = p_{gg}$). It must be that $\tau_g > \tau_b$ according to the optimality of $\{C_b, C_g\}$. Due to paper space limitation we don't give it a detailed proof.

Next we show that the IC constraint (3) holds as a strict inequality, that is, the property $(D.)$ satisfies. Suppose not, i.e. suppose the intermediate node did not strictly prefer C_b to C_g when he observes b, the end pair could then offer $\{C_g, C_g\}$ rather than $\{C_b, C_g\}$.$\{C_g, C_g\}$ is admissible in $(E.IP)$: First the node can attain his reservation utility. This is obvious if he observes g. And, since he weakly prefers C_g to C_b, he can also have at least reservation utility if he observes b. Second, the IC constraint (4) is trivially satisfied. But by $\tau_g > \tau_b$ and so the contract $\{C_b, C_g\}$ will strictly increase the end pair's expected profit. This is a contradiction.

The Proof of Proposition 5

First, note that $(A.)$,$(B.)$ and $(E.)$ in Proposition 2, together with the definition of $\bar{P}_b(k)$. $\eta > 0$ can be chosen sufficiently small so that

$$\begin{aligned} q_g(k)U(p_{gb}(k)+\eta) + (1-q_g(k))U(p_{gg}(k)) - D(t_g(k), g) \\ < q_g(k)U(\bar{p_b}(k)) + (1-q_g(k))U(p_{bb}(k)) - D(t_b(k), g)) \end{aligned} \tag{7}$$

$$U(p_{gb}(k)+\eta) - D(t_g(k), b) < \bar{U}(b) \tag{8}$$

Second, there are the following lemmas.

Lemma 1. *For $i = j, k$, if node i observes b, he never chooses to spend $t_g(i)$ on transferring message.*

Use $(B.)$ and $(D.)$ of Proposition 2, together with (8), for node $i = j, k$.

Lemma 2. *For each $\theta(0 < \theta < 1 - q_b(j))$, node j never randomizes between $t_b(j)(\theta)$ and $t_b(j)$, or between $t_b(j)(\theta)$ and $t_g(j)$, or between $t_b(j)(\theta)$ and refusing the contract.*

Suppose that this were not true when node j observes b or g. If he chooses some $t_b(j)(\theta)$ in b, where $0 < \theta < 1 - q_b(j)$, then under this condition, the probability that node k chooses $t_g(k)$ is lower than $1 - q_b(j)$. Otherwise, the probability is lower than $q_b(j)$. Together with $\triangle^\theta(q_b(j)) < 0$, we know that node j would strictly prefers choosing $t_b(j)$ to $t_b(j)(\theta)$. The arguments where node j chooses some $t_b(j)(\theta)$ in g are analogous. Hence, using $(A.)$, $(B.)$, $(C.)$, $(E.)$ of Proposition 2, we know that node j would strictly prefers choosing $t_b(j)$ to both $t_b(j)(\theta)$ and refusing the contract regardless of any states observed. This means that node j must be randomizing to choose between $t_b(j)$ and $t_b(j)(\theta)$. But (6) implies that he may strictly prefers $t_b(j)(\frac{\theta}{3})$. This is a contradiction.

Lemma 3. *If node j observes b, he never chooses to spend $t_b(j)$ on transferring message and also send a signal θ to the end pair, for each $\theta(0 < \theta < 1 - q_b(j))$.*

Suppose not. By Lemma 2, node j must always choose from the set $\{t_b(j)(\theta) \mid 0 < \theta < 1 - q_b(j)\}$. This means that the conditional probability that node k chooses $t_g(k)$ when node j observes b is lower than $1 - q_b(j)$, from Lemma 1, which implies the corresponding conditional probability when node j observes g is lower than $1 - q_g(j)$. Together with $(C.)$ and $(E.)$ of Proposition 2, it means that node j would strictly prefers choosing $t_b(j)$ to both $t_g(j)$ and refusing the contract. From Lemma 2, there are only two possibilities: either (a) node j always spends $t_b(j)$ on forwarding messages in g or (b) node j always chooses from the set $\{t_b(j)(\theta)|0 < \theta < 1 - q_b(j)\}$. Suppose that (a) is satisfied, then, from (7), node j would always chooses $t_b(j)$ when he observes g. But then node j never chooses $t_g(j)$, and so node j would strictly prefers some $t_b(j)(\theta)$ to $t_b(j)$ in g, a contradiction. Now suppose that (b) is satisfied, then by the definition of $\bar{P}_b(k)$ and $\eta > 0$, node j would strictly prefers $t_g(j)$ to both $t_b(j)$ and refusing the contract, when he observes g, a contradiction.

Lemma 4. *If node k observes g, he will choose $t_g(k)$ with certainty.*

Suppose not. Then $Prob$(node k chooses $t_b(k)$ |node j observes b) $> q_b(j)$. That is, there exists $\theta > 0$ such that $q > q_b(j)+\theta$. Thus node j should chooses $t_b(j)(\theta)$. This contradicts Lemma 3.

Lemma 5. *If node j observes g, he never chooses either to spend $t_b(j)$ on transferring message, or spend $t_b(j)$ on transferring message and also send a signal θ to the end pair.*

By Lemma 1 and 4, $Prob$(node k chooses $t_g(k)$ | node j observes g)$=1 - q_g(j)$. Hence node j would strictly prefers $t_b(j)$ to $t_b(j)(\theta)$, since $\triangle^\theta(q) < 0$ for $q \leq q_b(j)$ and $q_g(j) \leq q_b(j)$. If node j produces $t_b(j)$ in g, then from Lemma 1, $Prob$(node j chooses $t_g(j)|$ node k observes g)$< 1-q_g(k)$. Hence, by $(A.)$, $(E.)$ of Proposition 2, node k would strictly prefers $t_b(k)$ to $t_g(k)$ in g, which contradicts Lemma 4.

Lemma 6. *If node k observes b, he never refuses to sign the contract.*

Suppose node k refuses to sign the contract with some probability. Then by $(E.)$ of Proposition 2, we know that node j would strictly prefers $t_b(j)$ to $t_g(j)$ when he observes g, which contradicts Lemma 5.

Lemma 7. *Node j never refuses to sign the contract.*

The argument works is similar to Lemma 6.

Finally, by Lemma 1, 3, 5 and 7, we know that node j must choose transfer time in the light of the strategy sy^*. And by Lemma 1, 4 and 6, node k does likewise. Therefore, Proposition 5 satisfies.